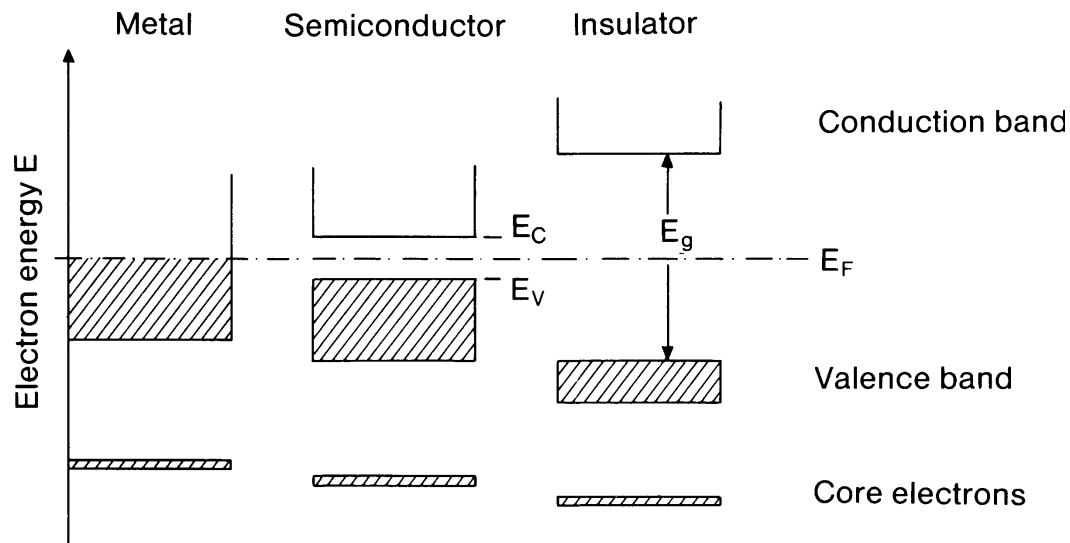


Semiconductors

- A. Formation of Band Gap
- B. Carrier Concentration
- C. Semiconductor conductivity
- D. *p-n* Junctions
- E. Biased *p-n* Junctions
- F. Schottky Contacts
- G. Heterostructures and Superlattices

Metal, Semiconductor and Insulator

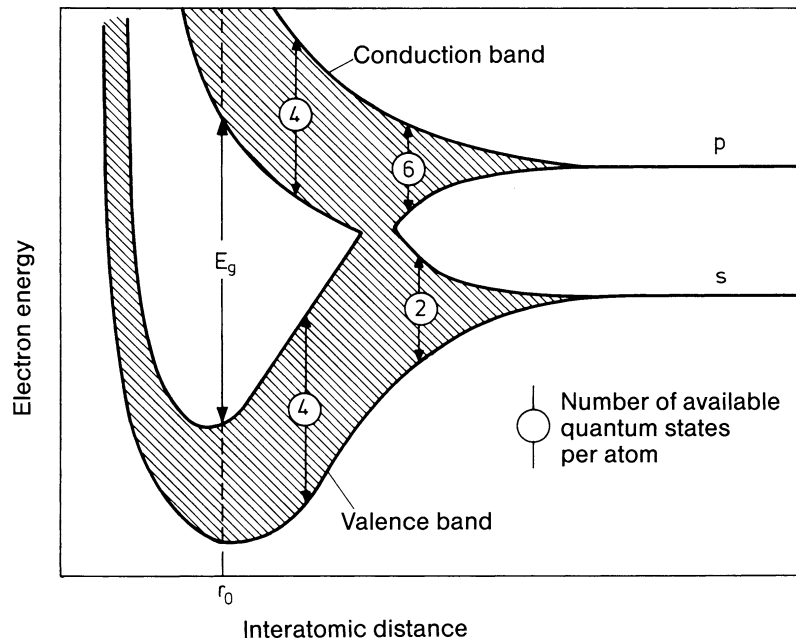
Completely filled bands and completely empty bands do not contribute to electrical conductivity, and only a partly filled electronic band can contribute to electric current.



If the distance between the upper edge of the highest filled band (valence band) and the lower edge of the lowest empty band (conduction band) is not too large (e.g. ~ 1 eV), then a small fraction of the states in the vicinity of the upper edge of the valence band is unoccupied and the corresponding electrons are found in the conduction band.

Formation of Energy Bands

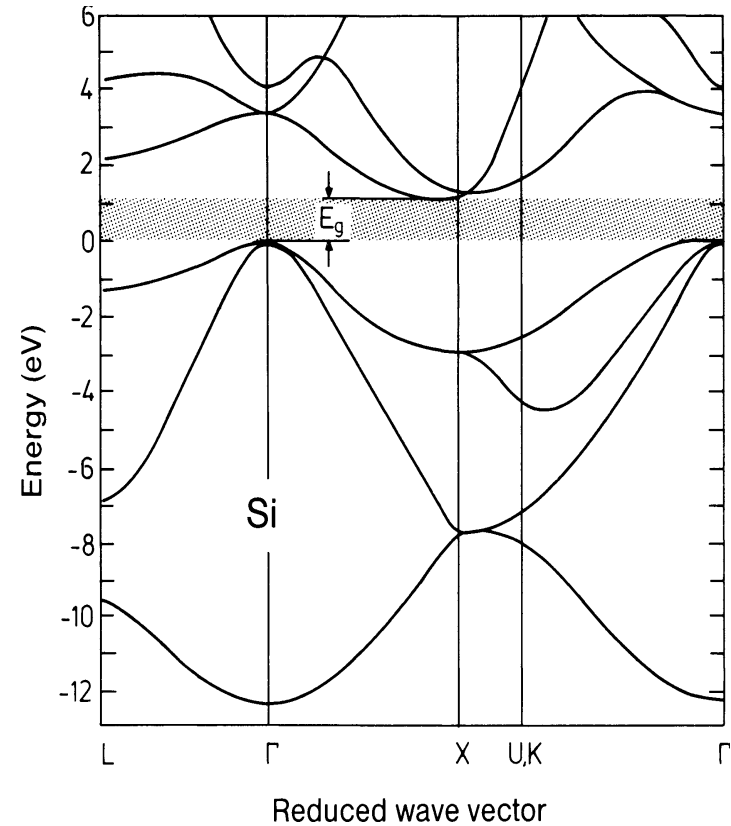
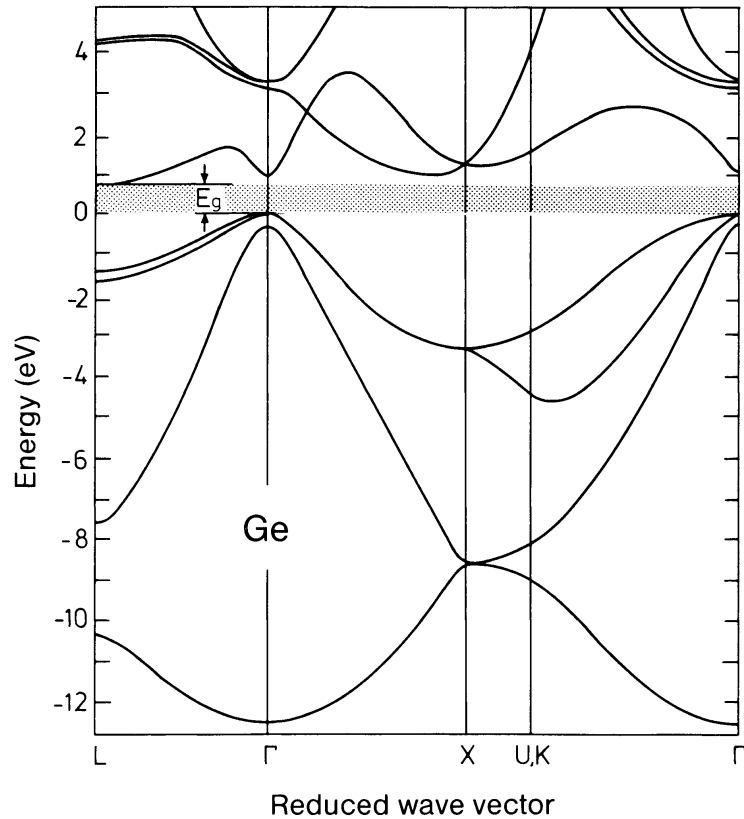
The origin of the band structure for the typical elemental semiconductors is due to a mixing of the s- and p-wavefunctions, tetrahedral bonding orbitals (sp^3) are formed. The bonding orbitals constitute the valence band and the antibonding orbitals the conduction band. The size of the band gap must be temperature dependent. With increasing temperature the lattice parameter increases due to thermal expansion. The splitting between the bonding and antibonding states therefore decreases and the band gap becomes smaller.



Energy gaps of Si and Ge

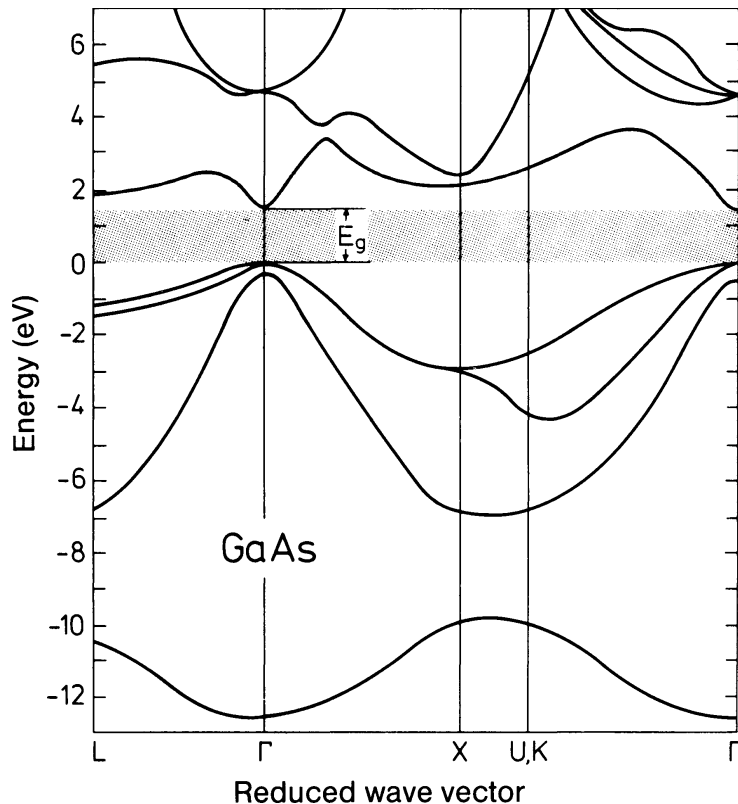
	$E_g (T = 0 \text{ K})$ (eV)	$E_g (T = 300 \text{ K})$ (eV)
Si	1.17	1.1
Ge	0.75	0.67

Energy Bands of Germanium and Silicon



Compound Semiconductors

Based on the sp^3 bonding, one can correctly identify another important class, the III-V semiconductors, which are compound semiconductors comprising elements from the third and fifth groups of the periodic table. Typical examples are InSb, InAs, InP, GaP, GaAs, GaSb, and AlSb. In these compound crystals the bonding is mixed ionic and covalent .



In contrast to the elemental semiconductors, the most important representatives of the III-V semiconductors possess a so-called direct band gap.

	$E_g(0\text{ K})$ [eV]	$E_g(300\text{ K})$ [eV]	m_n^*/m	m_{lh}^*/m	m_{hh}^*/m	m_{soh}^*/m	Δ [eV]
GaAs	1.52	1.43	0.07	0.08	0.5	0.15	0.34
GaSb	0.81	0.7	0.047	0.05	0.3	0.14	0.8
InSb	0.24	0.18	0.015	0.02	0.4	0.11	0.8
InAs	0.43	0.35	0.026	0.025	0.4	0.14	0.4
InP	1.42	1.35	0.073	0.12	0.6	0.12	0.11

Intrinsic Carrier Concentrations

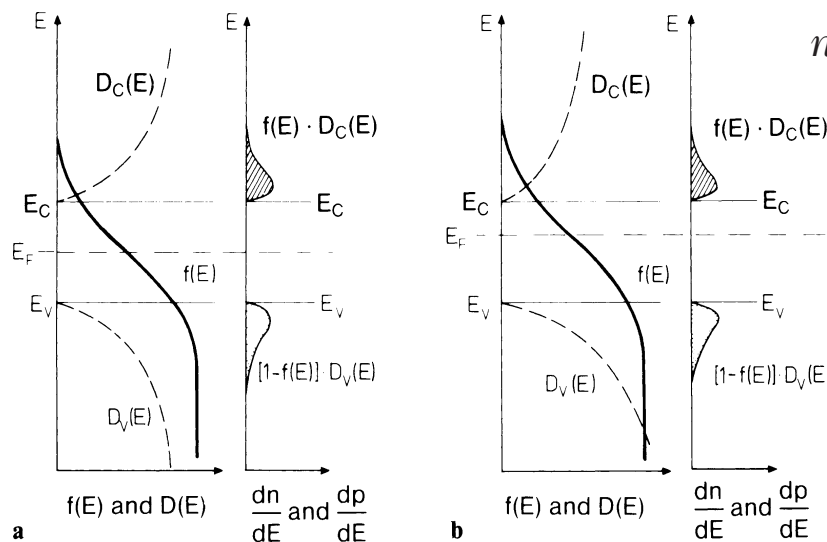
For a semiconductor, we want to know the concentration of intrinsic carries as function of temperature T , in terms of its band gap E_g .

The energy of an electron in the conduction band is $\epsilon_k = E_c + \hbar^2 k^2 / 2m_e$

The density of electron states at ϵ is $D_e(\epsilon) = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} (\epsilon - E_c)^{1/2}$

Fermi-Dirac distribution for $\epsilon - \mu \gg k_B T$ reduces to $f_e \approx \exp[(\mu - \epsilon) / k_B T]$

The concentration of electrons in the conduction band is



$$n = \int_{E_c}^{\infty} D_e(\epsilon) f_e(\epsilon) d\epsilon = \frac{1}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} \exp(\mu/k_B T) \times \int_{E_c}^{\infty} (\epsilon - E_c)^{1/2} \exp(-\epsilon/k_B T) d\epsilon ,$$

which integrates to give

$$n = 2 \left(\frac{m_e k_B T}{2\pi \hbar^2} \right)^{3/2} \exp[(\mu - E_c) / k_B T]$$

The distribution function f_h for holes is related to the electron distribution by

$$f_h = 1 - \frac{1}{\exp[(\epsilon - \mu)/k_B T] + 1} = \frac{1}{\exp[(\mu - \epsilon)/k_B T] + 1}$$
$$\cong \exp[(\epsilon - \mu)/k_B T] , \quad \text{if } (\mu - \epsilon) \gg k_B T.$$

The density of hole states at ϵ is

$$D_h(\epsilon) = \frac{1}{2\pi^2} \left(\frac{2m_h}{\hbar^2} \right)^{3/2} (E_v - \epsilon)^{1/2} ,$$

The concentration p of holes in the valence band is

$$p = \int_{-\infty}^{E_c} D_h(\epsilon) f_h(\epsilon) d\epsilon = 2 \left(\frac{m_h k_B T}{2\pi \hbar^2} \right)^{3/2} \exp[(E_v - \mu)/k_B T]$$

We multiply together the expressions for n and p to obtain the equilibrium relation

$$np = 4 \left(\frac{k_B T}{2\pi \hbar^2} \right)^3 (m_c m_h)^{3/2} \exp(-E_g/k_B T) , \quad E_g = E_c - E_v$$

The product of np is constant at a given temperature.

For intrinsic carriers, n_i and p_i ,

$$n_i = p_i = 2 \left(\frac{k_B T}{2\pi\hbar^2} \right)^{3/2} (m_e m_h)^{3/4} \exp(-E_g/2k_B T) .$$

To obtain the Fermi level μ , we start from $n_i = p_i$, so

$$n_i = 2 \left(\frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2} \exp[(\mu - E_c)/k_B T] = 2 \left(\frac{m_h k_B T}{2\pi\hbar^2} \right)^{3/2} \exp[(E_c - \mu)/k_B T] = p_i$$

then,

$$\exp(2\mu/k_B T) = (m_h/m_e)^{3/2} \exp(E_g/k_B T) ;$$

$$\mu = \frac{1}{2} E_g + \frac{3}{4} k_B T \ln (m_h/m_e) .$$

If $m_h = m_e$, then $\mu = 1/2 E_g$ and the Fermi level is in the middle of the forbidden gap.

Impurity Carrier Concentrations

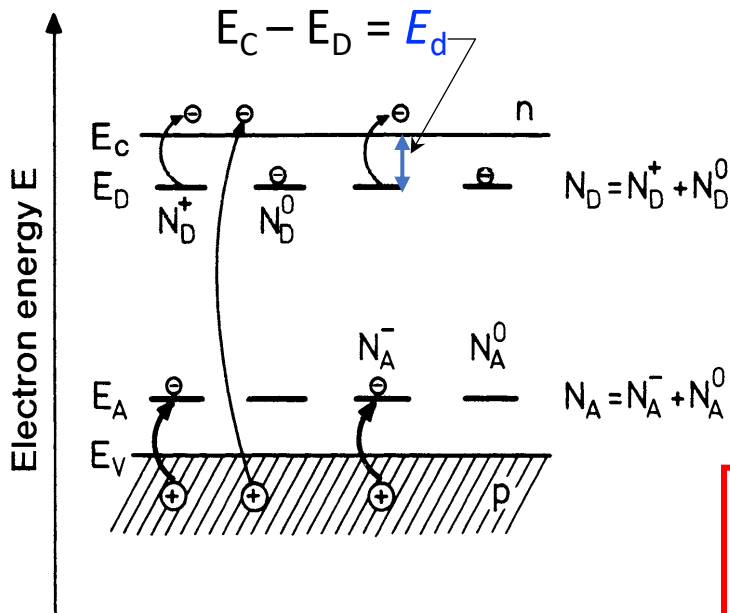
For a doped semiconductor, we want to know the concentration of conductive carries as function of temperature T and impurity energy levels E_d and E_a .

Neutrality condition demands $n + N_A^- = p + N_D^+$, in which

$$N_D = N_D^0 + N_D^+, \quad \text{and} \quad n_D = N_D^0 = N_D [1 + \exp(E_D - E_F)/kT]^{-1},$$

$$N_A = N_A^0 + N_A^-. \quad p_A = N_A^0 = N_A [1 + \exp(E_F - E_A)/kT]^{-1}.$$

For the case of a pure n-type semiconductor, with $N_D^+ \gg n_i$,



$$n \approx N_D^+ = N_D - N_D^0$$

$$\approx N_D \left(1 - \frac{1}{1 + \exp[(E_D - E_F)/kT]} \right).$$

Since $(n/N_{\text{eff}}^C) e^{E_C/kT} = e^{E_F/kT}$,

$$n \approx \sqrt{N_D N_{\text{eff}}^C} e^{-E_d/2kT}$$

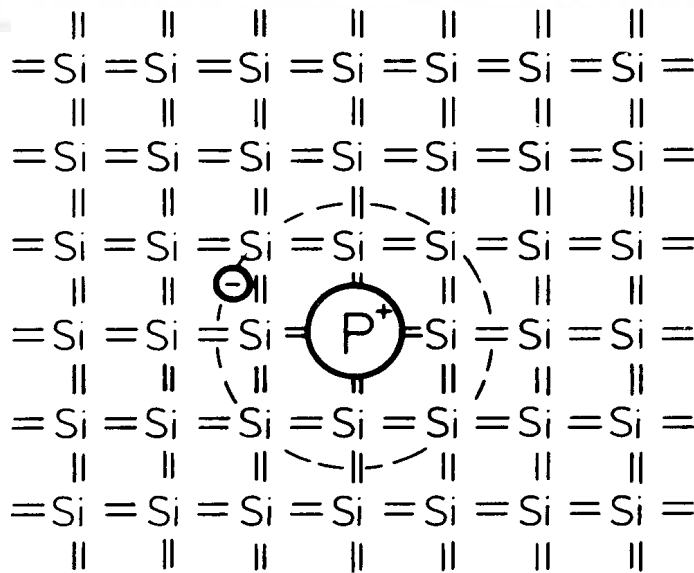
Impurity Conductivity

Certain impurities and imperfections drastically affect the electrical properties of a semiconductor. The addition of boron to silicon in the proportion of 1 boron atom to 10^5 silicon atoms increases the conductivity of pure at room temperature by a factor of 10^3 . The deliberate addition of impurities to a semiconductor is called **doping**.

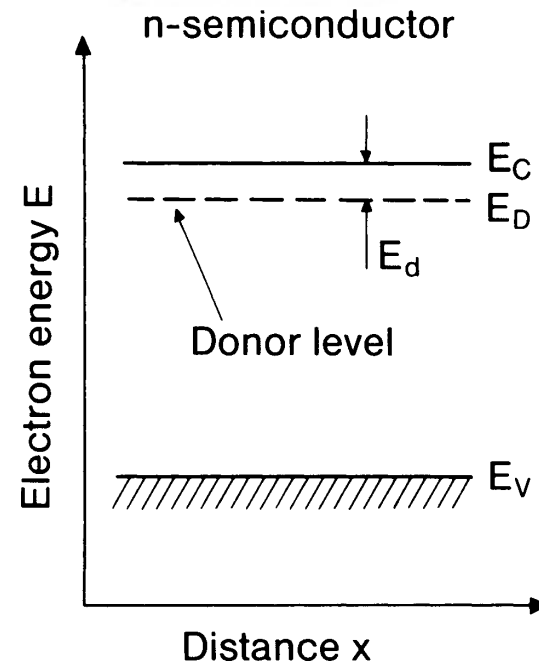
We consider the effect of impurities in silicon and germanium. These elements crystallize in the diamond structure. Each atom forms four covalent bonds, one with each of its nearest neighbors, corresponding to the chemical valence four. If an impurity atom of valence five, such as phosphorus, arsenic, or antimony, is substituted in the lattice in place of a normal atom, there will be one valence electron from the impurity atom left over after the four covalent bonds are established with the nearest neighbors, that is, after the impurity atom has been accommodated in the structure with as little disturbance as possible. Impurity atoms that can give up an electron are called **donors**.

Donor States

The impurity atoms of valence five such as P, As, and Sb are called **donors** because they donate electrons to the conduction band in order to complete the covalent bonds with neighbor atoms, leaving electrons in the band.



a n-doped silicon



The extra electron moves in the coulomb potential $e/\epsilon r$ of the impurity ion, where ϵ in a covalent crystal is the static dielectric constant of the medium.

We estimate the ionization energy of the donor impurity. The Bohr theory of the hydrogen atom may be modified to take into account the dielectric constant of the medium and the effective mass of an electron in the periodic potential of the crystal. The ionization energy of atomic hydrogen is $-e^4m/(2\hbar^2)$.

In the semiconductor with dielectric constant ϵ we replace e^2 by e^2/ϵ and m by the effective mass m_e to obtain

$$E_d = \frac{e^4 m_e}{2\epsilon^2 \hbar^2} = \left(\frac{13.6 m_e}{\epsilon^2 m} \right) eV ; \quad \text{and} \quad a_d = \frac{\epsilon \hbar^2}{m_e e^2} = \left(\frac{0.53 \epsilon}{m_e/m} \right) \text{\AA}$$

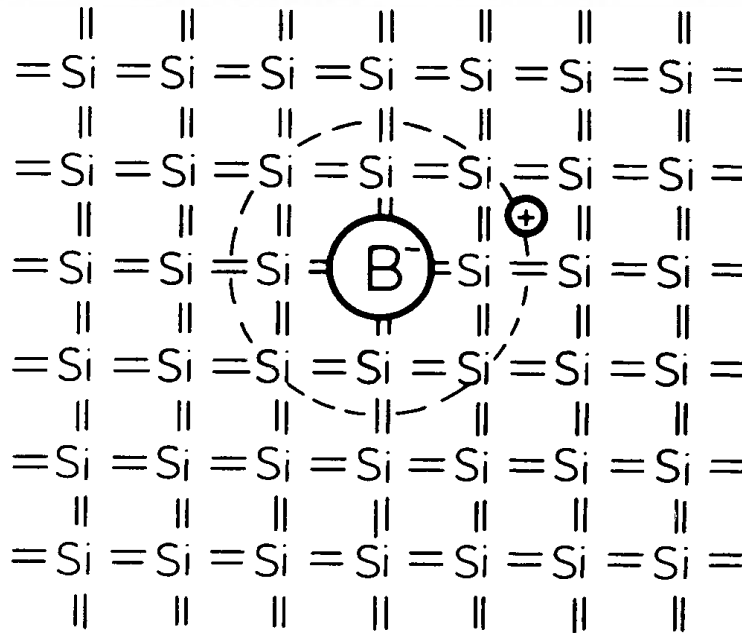
E_d is the ionization energy and a_d the Bohr radius of the donor.

To obtain a general impression of the impurity levels we use $m_e \simeq 0.1 m$ for electrons in germanium and $m_e \simeq 0.2 m$ in silicon. The static dielectric constant ϵ is 15.8 for Ge and 11.7 for Si. Then, we obtain

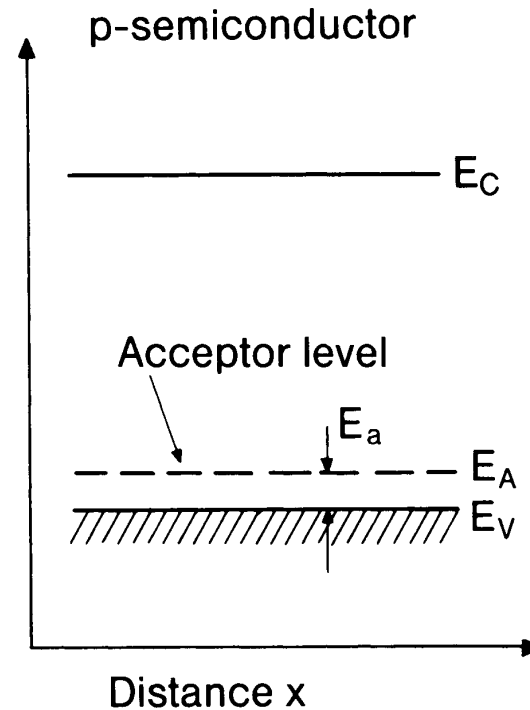
$$E_d = 5 \text{ meV} \quad \text{and} \quad a_d = 80 \text{ \AA} \quad \text{for Ge;}$$

$$E_d = 20 \text{ meV} \quad \text{and} \quad a_d = 30 \text{ \AA} \quad \text{for Si.}$$

Acceptor States



b p-doped silicon



Trivalent impurities such as B, Al, Ga, and In are called **acceptors** because they accept electrons from the valence band in order to complete the covalent bonds with neighbor atoms, leaving holes in the band.

Properties of Holes

Holes are vacant orbits in a band whose properties are important in an almost filled band.

A hole acts in applied electric and magnetic fields as if it has a positive charge $+e$. The reason is given in five steps that follow:

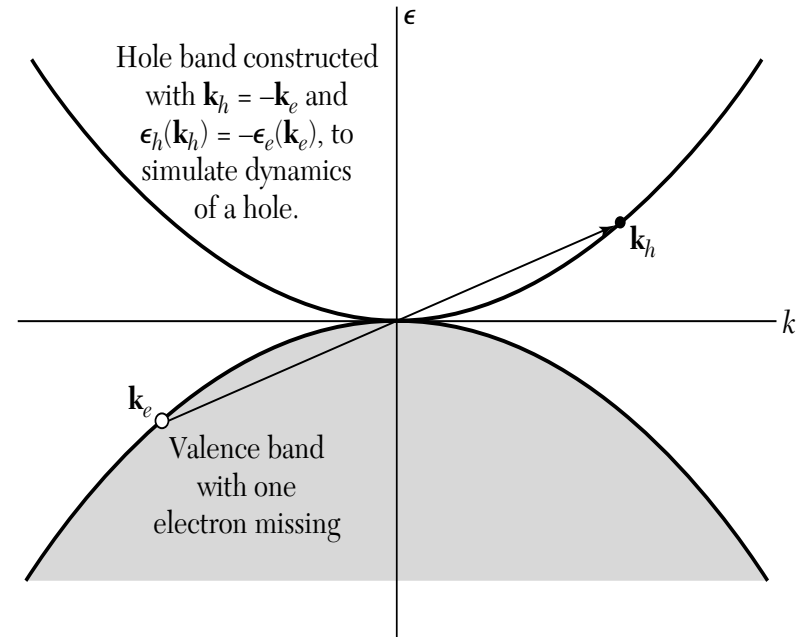
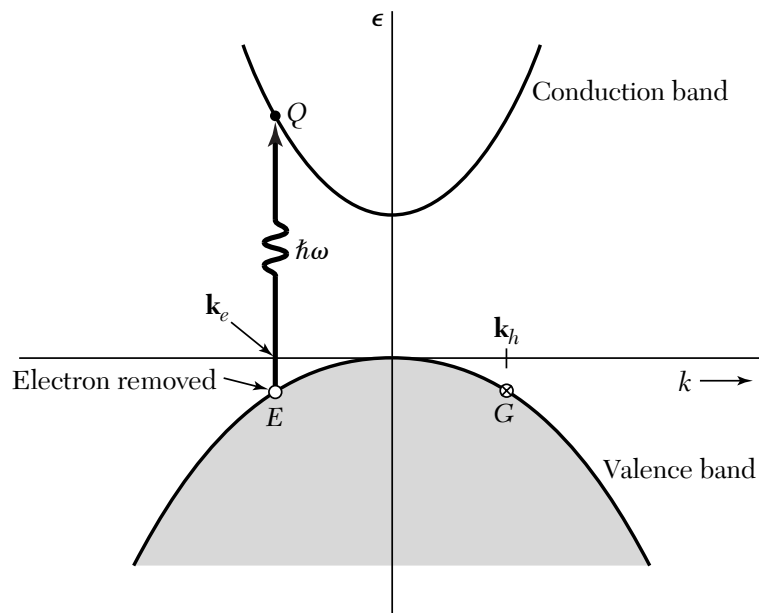
1. Crystal momentum: $\mathbf{k}_h = -\mathbf{k}_e$

The total wavevector of the electrons in a filled band is zero: $\sum \mathbf{k} = 0$, where the sum is over all states in a Brillouin zone. This result follows from the geometrical symmetry of the Brillouin zone: every fundamental lattice type has symmetry under the inversion operation $\mathbf{r} \rightarrow -\mathbf{r}$ about any lattice point; it follows that the Brillouin zone of the lattice also has inversion symmetry. If the band is filled all pairs of orbitals \mathbf{k} and $-\mathbf{k}$ are filled, and the total wavevector is zero.

If an electron is missing from an orbital of wavevector \mathbf{k}_e , the total wavevector of the system is $-\mathbf{k}_e$ and is attributed to the hole.

2. Energy: $\epsilon_h(\mathbf{k}_h) = -\epsilon_e(\mathbf{k}_e)$

The energy of the hole is opposite in sign to the energy of the missing electron, because it takes more work to remove an electron from a low orbital than from a high orbital.



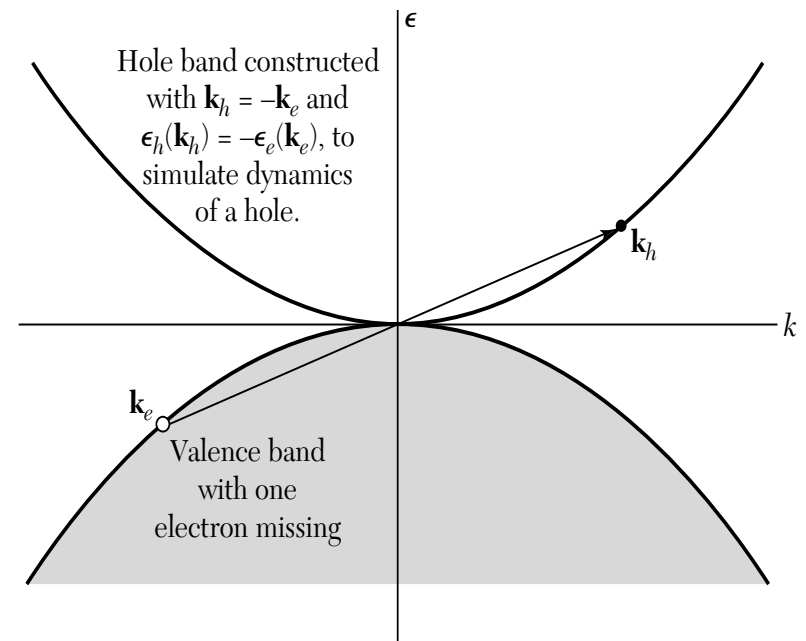
3. Velocity: $\mathbf{v}_h = \mathbf{v}_e$

The velocity of the hole is equal to the velocity of the missing electron.

Since $\nabla \epsilon_h(\mathbf{k}_h) = \nabla \epsilon_e(\mathbf{k}_e)$, so that $\mathbf{v}_h(\mathbf{k}_h) = \mathbf{v}_e(\mathbf{k}_e)$.

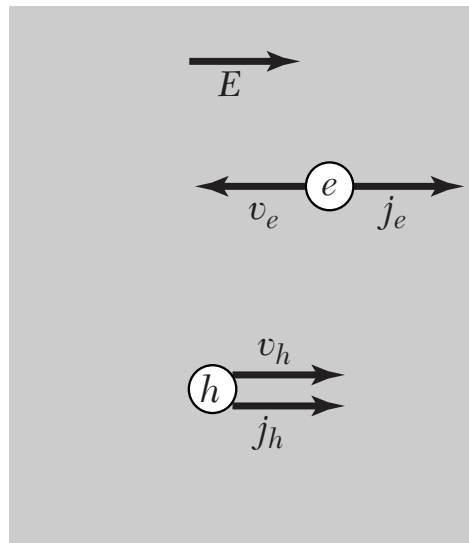
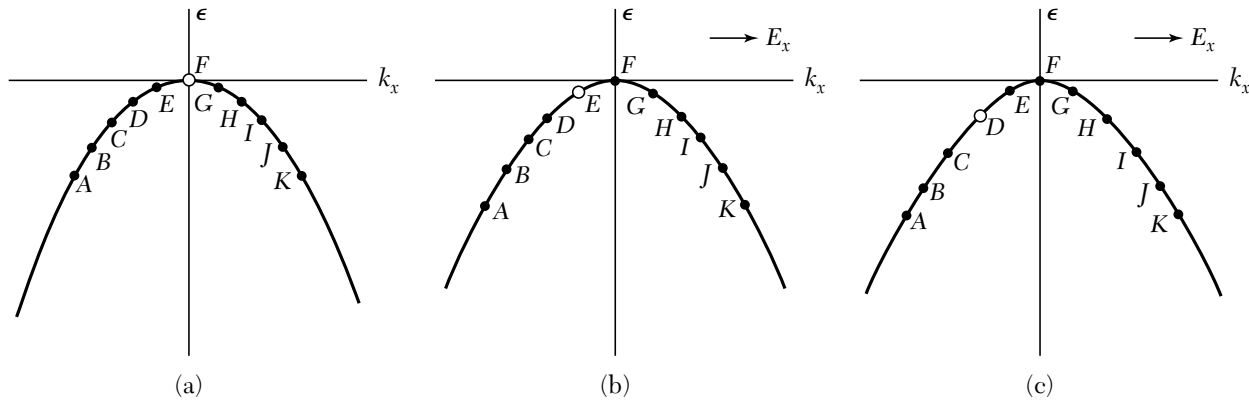
4. Effective mass: $m_h = -m_e$

The effective mass is inversely proportional to the curvature $d^2\epsilon/dk^2$, and for the hole band this has the opposite sign to that for an electron in the valence band.



5. Equation of motion:
$$\hbar \frac{d\mathbf{k}_h}{dt} = e(\mathbf{E} + \frac{1}{c} \mathbf{v}_h \times \mathbf{B})$$

The equation of motion for a hole is that of a particle of positive charge e .



Electrons and Holes

A full band can be written as: $\sum_{i \neq +j}^{\pm N/2} \vec{k}_i + \vec{k}_j = 0$ and $\sum_{i \neq +j}^{\pm N/2} \vec{k}_i = -\vec{k}_j = \vec{k}_{-j}$

So when the state +j is empty in a band, the band has effective wavevector k_{-j} .

Now the current flow in the incomplete band under the influence of a field

E:

$$\sum_{i=\pm 1}^{\pm N/2} (-e\vec{v}_i) = \underbrace{\sum_{i \neq +j}^{\pm N/2} (-e\vec{v}_i)}_{e\vec{v}_j} - e\vec{v}_j = 0$$

This shows that an incomplete band (state +j empty) behaves just like a positive charge moving with the same velocity an electron would have in that state. Thus the properties of all of the remaining electrons in the incomplete band are equivalent to those of the vacant state j if the vacant state has:

- A k-vector k_{-j}
- A velocity v_{+j}
- A positive charge $+e$

We call this vacant state a positive "hole" (h^+)

Dynamics of Electrons and Holes

If this hole is accelerated in an applied electric field: $m_h \frac{d\vec{v}_h}{dt} = +e\vec{E}$

The corresponding equation for the electron is: $m_e \frac{d\vec{v}_e}{dt} = -e\vec{E}$

But earlier we deduced that the hole velocity is the same as that of the corresponding “missing” electron: $\vec{v}_h = \vec{v}_e$

So by equating the derivatives we find: $\frac{e\vec{E}}{m_h} = -\frac{e\vec{E}}{m_e} \longrightarrow m_h = -m_e$

However, note that near the top of a band the band curvature is negative, so the effective electron mass is also negative. The corresponding hole mass is then positive.

So the equation of motion of a “hole” in an electromagnetic field is:

$$\vec{F} = \hbar \frac{d\vec{k}_h}{dt} = e(\vec{E} + \vec{v}_h \times \vec{B})$$

This explains why some metals have positive R_H .

Carrier Mobility

The mobility is the magnitude of the drift velocity of a charge carrier per unit electric field:

$$\mu = |v|/E .$$

The electrical conductivity is the sum of the electron and hole contributions:

$$\sigma = (ne\mu_e + pe\mu_h)$$

The drift velocity of a charge q was found to be $v = q\tau E/m$, whence

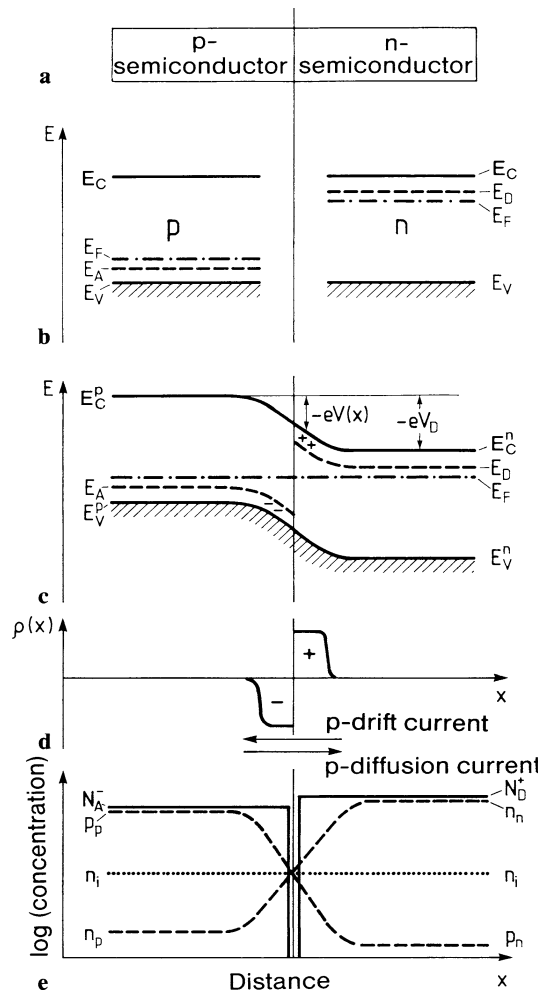
$$\mu_e = e\tau_e/m_e ; \quad \mu_h = e\tau_h/m_h , \quad \text{where } \tau \text{ is the collision time.}$$

Table 3 Carrier mobilities at room temperature, in cm²/V-s

Crystal	Electrons	Holes	Crystal	Electrons	Holes
Diamond	1800	1200	GaAs	8000	300
Si	1350	480	GaSb	5000	1000
Ge	3600	1800	PbS	550	600
InSb	800	450	PbSe	1020	930
InAs	30000	450	PbTe	2500	1000
InP	4500	100	AgCl	50	—
AlAs	280	—	KBr (100 K)	100	—
AlSb	900	400	SiC	100	10–20

p-n Junction

The most important building blocks in semiconductor devices are the *p-n* junction and the metal/semiconductor contact. In a *p-n* junction, we have a semiconductor crystal which is *p*-type on one side, and *n*-type on the other.



In the transition zone between the *n* and *p* regions, the Fermi level, i.e. the electrochemical potential, must therefore be common across the junction at thermal equilibrium, which therefore causes the band bending, as shown in the left panel **c**. The potential $V(x)$ changes near the junction and, according to the Poisson equation, the macro-potential $V(x)$ corresponds to a space charge $\rho(x)$

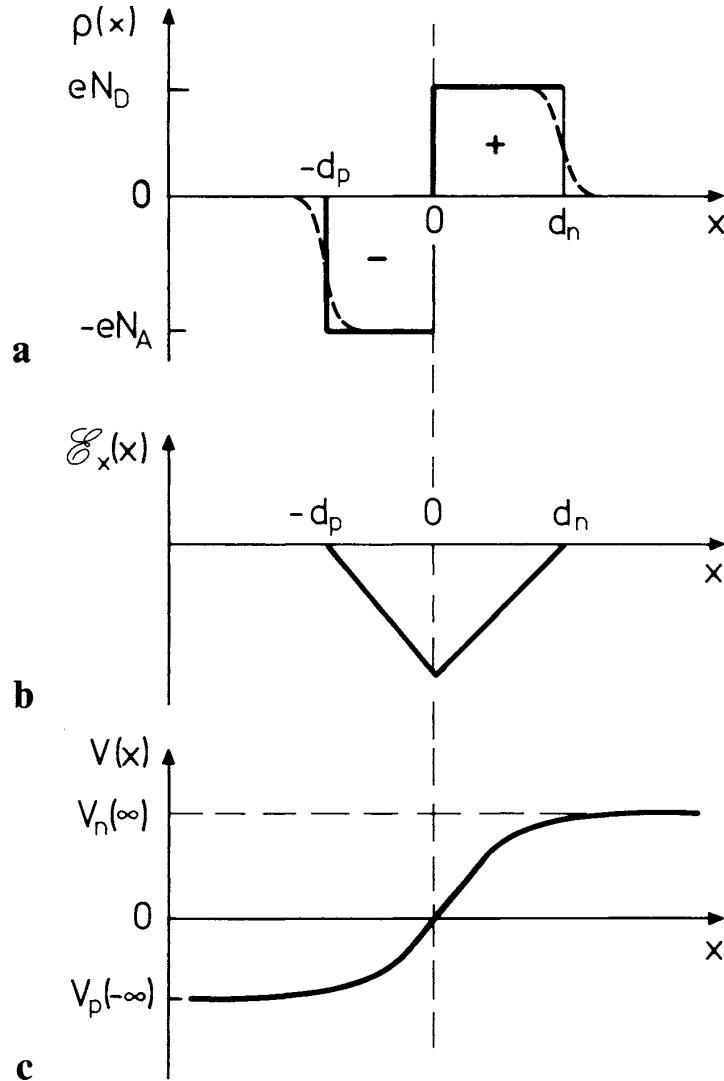
$$\frac{\partial^2 V(x)}{\partial x^2} = -\frac{\rho(x)}{\epsilon \epsilon_0}.$$

The diffusion voltage (V_D) is the difference between the maximum and minimum of $V(x)$:

$$eV_D = -(E_V^n - E_V^p) = kT \ln \frac{p_p n_n}{n_i^2}.$$

Assume the space-charge density is a step function:

$$\rho(x) = \begin{cases} 0 & \text{for } x < -d_p \\ -eN_A & \text{for } -d_p < x < 0 \\ eN_D & \text{for } 0 < x < d_n \\ 0 & \text{for } x > d_n \end{cases}$$



The Poisson equation, for the n region ($0 < x < d_n$)

$$\frac{d^2 V(x)}{dx^2} \simeq -\frac{eN_D}{\epsilon \epsilon_0}$$

The electric field in n region is

$$\mathcal{E}_x(x) = -\frac{e}{\epsilon \epsilon_0} N_D (d_n - x)$$

and the potential in n region is

$$V(x) = V_n(\infty) - \frac{eN_D}{2\epsilon \epsilon_0} (d_n - x)^2$$

and the potential in p region is

$$V(x) = V_p(-\infty) + (eN_A/2\epsilon\epsilon_0)(x - d_p)^2$$

Within the Schottky model, the lengths d_n and d_p give the spatial extent of the space-charge zone in the n and p regions, respectively. From charge neutrality it follows that

$$N_D d_n = N_A d_p ,$$

and the continuity of $V(x)$ at $x = 0$ demands

$$\frac{e}{2 \varepsilon \varepsilon_0} (N_D d_n^2 + N_A d_p^2) = V_n(\infty) - V_p(-\infty) = V_D .$$

If the impurity concentrations are known, one can thus calculate the spatial extent of the space-charge layer

$$d_n = \left(\frac{2 \varepsilon \varepsilon_0 V_D}{e} \frac{N_A / N_D}{N_A + N_D} \right)^{1/2} ,$$

$$d_p = \left(\frac{2 \varepsilon \varepsilon_0 V_D}{e} \frac{N_D / N_A}{N_A + N_D} \right)^{1/2} .$$

Biased p - n Junction

If an external electrical voltage U is applied to a p - n junction, thermal equilibrium is destroyed, and the situation in the p - n junction can be described as a stationary state in the vicinity of thermal equilibrium. Because the space-charge zone between $-d_p$ and d_n has a considerably higher electrical resistance than the region outside the p - n junction, the potential drop across the space-charge zone accounts for nearly all of the externally applied voltage U . Thus the potential drop across the space-charge zone, instead of being equal to the diffusion voltage V_D , now has the value

$$V_n(\infty) - V_p(-\infty) = V_D - U .$$

One thus has

$$d_n(U) = d_n(U = 0)(1 - U/V_D)^{1/2} ,$$
$$d_p(U) = d_p(U = 0)(1 - U/V_D)^{1/2} .$$

Let us consider the balance in the electron currents: we are concerned on the one hand with the drift currents of the minority carriers coming from the p region (where electrons are the minority carriers), which are drawn across into the n region. Because these minority carriers are continually generated in the p region by thermal excitation, this current is called the *generation current*, $I_n^{\text{gen}} = N_c^{\text{eff}} e^{-E_g/2kT}$.

This current is largely independent of the value of the diffusion voltage and therefore also of the external voltage.

The diffusion current of electrons from the n region, where the electrons are majority carriers, into the p region (called the *recombination current* I_n^{rec}). With an externally applied voltage U :

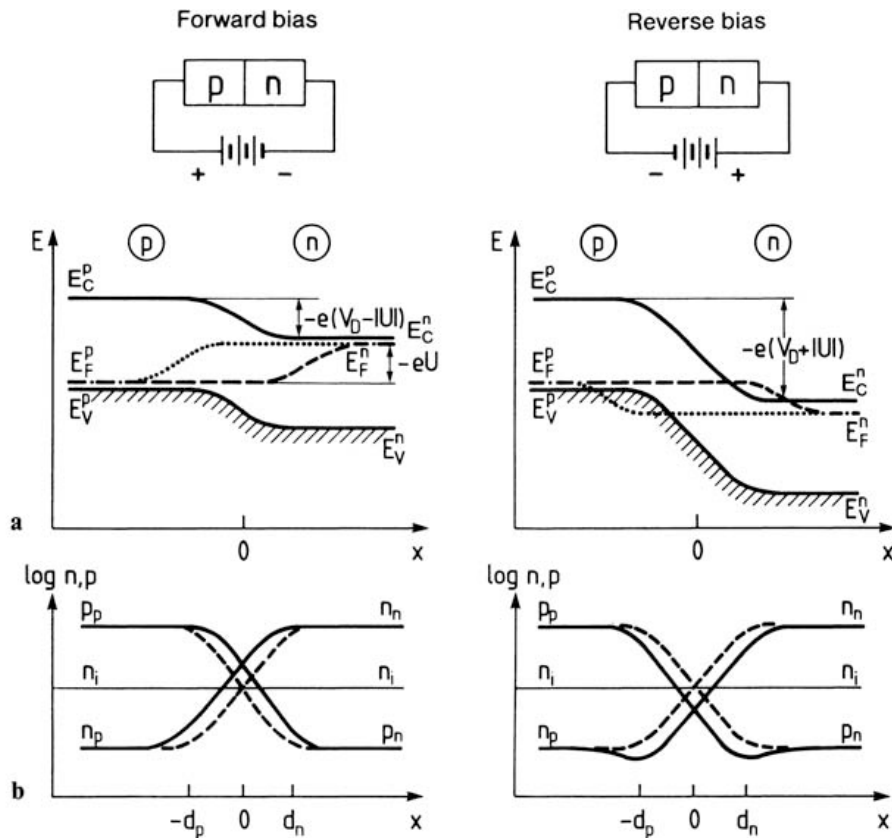
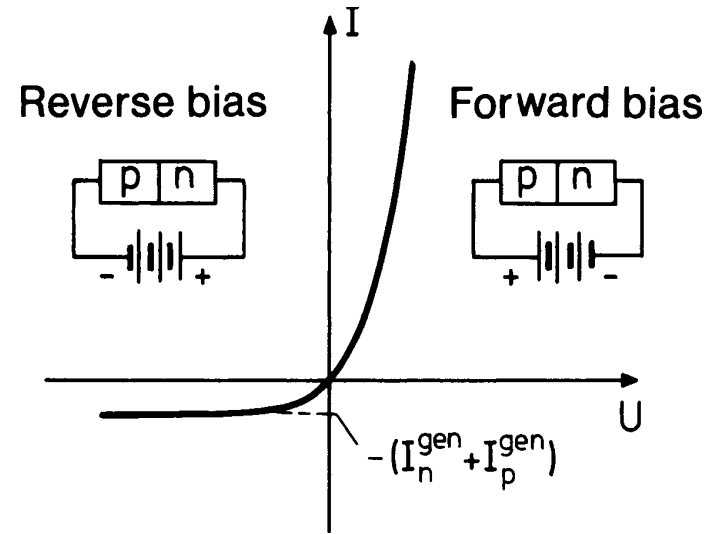
$$I_n^{\text{rec}} \propto e^{-e(V_D - U)/kT} = I_n^{\text{gen}} e^{eU/kT},$$

and therefore a total electron current I_n is

$$I_n = I_n^{\text{rec}} - I_n^{\text{gen}} = I_n^{\text{gen}} (e^{eU/kT} - 1).$$

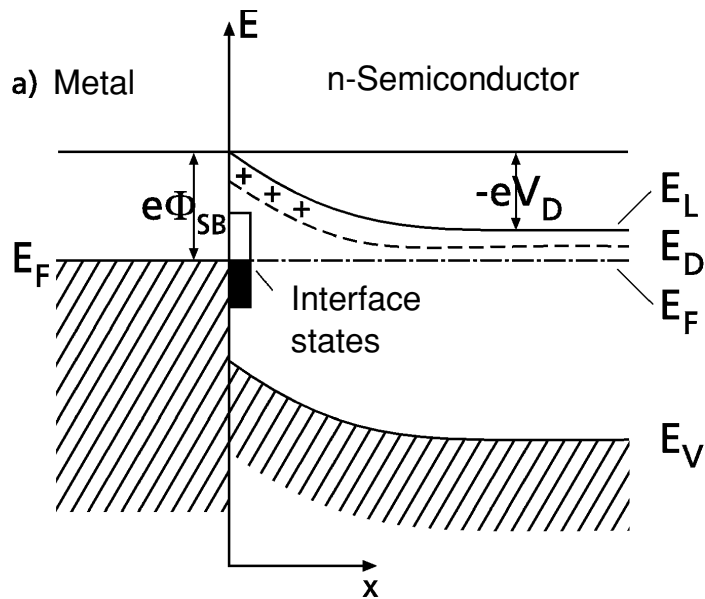
The total current $I(U)$ under applied external U is

$$I(U) = (I_n^{\text{gen}} + I_p^{\text{gen}}) \left(\exp \frac{eU}{kT} - 1 \right).$$



Metal/Semiconductor Schottky Contact

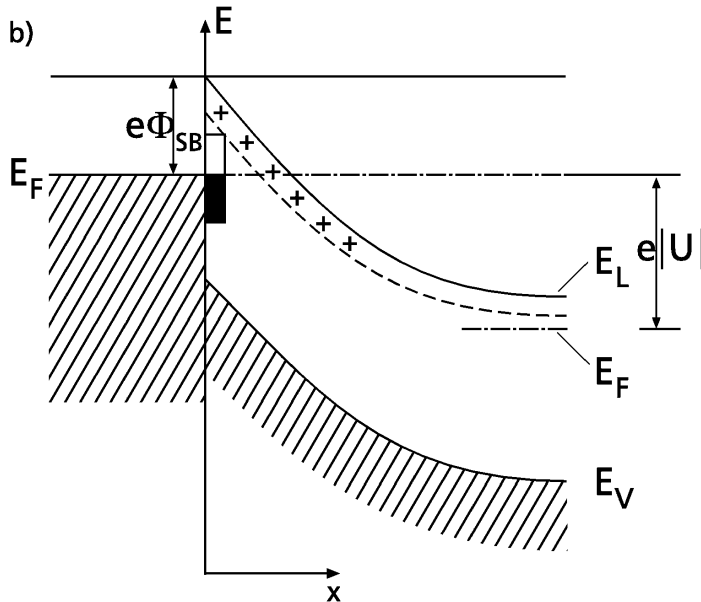
When a metal is evaporated onto a clean n -type semiconductor surface under good vacuum conditions, mostly an electronic band scheme as below.



Electronic interface states are formed at the metal and semiconductor interface. Their spatial extension is limited to a few atomic layers at the interface and their energetic distribution is fixed with respect to the conduction and valence band edges of the semiconductor. These interface states, called MIGS (metal-induced gap states) originate from the Bloch waves in the metal.

Conduction band states are negatively charged when occupied by electrons and neutral in the unoccupied state (acceptor-like); valence band states, however have a donor-like charging character, i.e. positive when empty and neutral in the occupied state.

The band bending within the semiconductor is thus determined by a charge balance between negative charge Q_{is} in the interface states and the positive space charge Q_{SC} in the depletion layer. An external bias produces a voltage drop essentially across the space-charge zone.



The mathematical description of the space-charge region below a metal-semiconductor junction is analogous to a $p-n$ junction. The thickness of the Schottky contact space charge region in thermal equilibrium, e.g., is obtained as

$$d = \left(\frac{2 \varepsilon \varepsilon_0 V_D}{e N_D} \right)^{1/2}$$

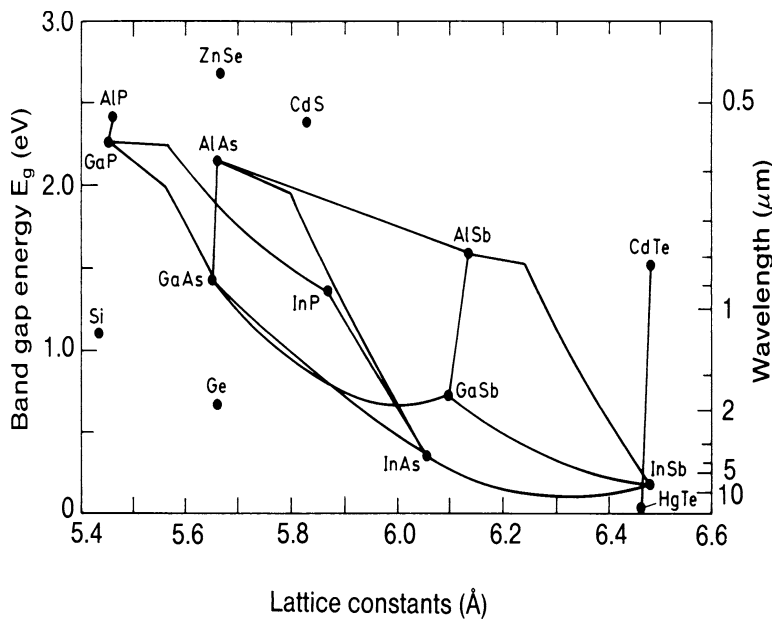
Similarly, the capacity of a metal-semiconductor junction as a function of external bias can be obtained as

$$C = \frac{A}{2} \left(\frac{2 e \varepsilon \varepsilon_0 N_D}{V_D - U} \right)^{1/2},$$

where A is the area of the contact. Electron transport from the metal into the semiconductor requires that the carriers overcome the Schottky barrier $e\Phi_{SB}$.

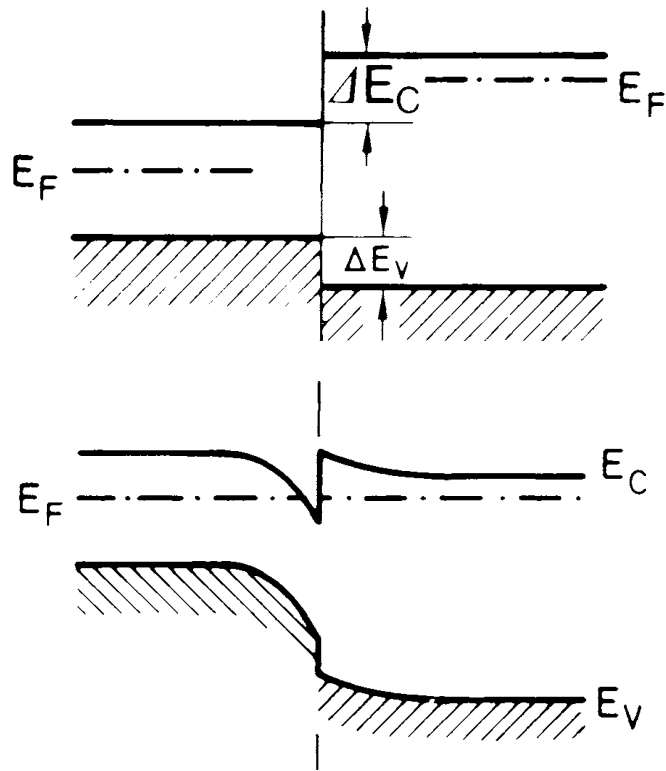
Semiconductor Heterostructures

The structure consisting of layers of two different semiconductors grown epitaxially on one another is called a semiconductor heterostructure. Using modern epitaxial methods, such as molecular beam epitaxy (MBE) or metal organic chemical vapor deposition (MOCVD), it is possible to deposit two different semiconductors on one another in a crystalline form. It is also significant that using such epitaxial methods, ternary and quaternary alloys of the type $\text{Al}_x\text{Ga}_{1-x}\text{As}$ or $\text{Ga}_x\text{In}_{1-x}\text{As}_y\text{P}_{1-y}$ can be deposited, whose band gaps lie between those of the corresponding binary compounds.



It is possible to produce particularly good and defect-free epitaxial layers from semiconductors whose lattice constants match. It is expected that the two components GaAs and AlAs should lead to excellent crystalline quality with a variation of the band gap between 1.4 and 2.2 eV.

In a heterostructure the band gap changes over distances of atomic dimensions. Two important points need to be considered for the electronic band structure of such a semiconductor heterostructure:



a) How are the valence band edges E_V and conduction band edges E_C to be "lined up"? This question addresses the so-called band discontinuity or band offset ΔE_V .

b) What band bending occurs in the two semiconductors I and II to the left and right of the junction?

The most important material-related parameters of a semiconductor heterostructure are therefore the valence and conduction band discontinuities, ΔE_V and ΔE_C .

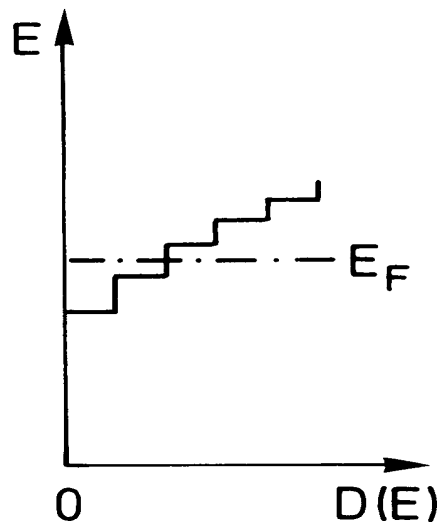
Band Offsets of Semiconductor Heterostructures

Band offsets can well be explained by models in which the electronic bands in ideal, abrupt semiconductor heterostructures are lined up so that no atomic dipoles are created, due to electronic interface states or charge transfer in the chemical bonds at the interface. A detailed theoretical treatment of these models requires a microscopic description of the electronic properties of the few atomic layers at the semiconductor junction. A few experimentally determined valence band discontinuities ΔE_V are listed below:

Hetero-structure	Valence band discontinuity ΔE_V [eV]	Hetero-structure	Valence band discontinuity ΔE_V [eV]	Hetero-structure	Valence band discontinuity ΔE_V [eV]
Si-Ge	0.28	InAs-Ge	0.33	CdTe- α -Sn	1.1
AlAs-Ge	0.86	InAs-Si	0.15	ZnSe-Ge	1.40
AlAs-GaAs	0.34	InP-Ge	0.64	ZnSe-Si	1.25
AlSb-GaSb	0.4	InP-Si	0.57	ZnSe-GaAs	1.03
GaAs-Ge	0.49	InSb-Ge	0.0	ZnTe-Ge	0.95
GaAs-Si	0.05	InSb-Si	0.0	ZnTe-Si	0.85
GaAs-InAs	0.17	CdS-Ge	1.75	GaSe-Ge	0.83
GaP-Ge	0.80	CdS-Si	1.55	GaSe-Si	0.74
GaP-Si	0.80	CdSe-Ge	1.30	CuBr-GaAs	0.85
GaSb-Ge	0.20	CdSe-Si	1.20	CuBr-Ge	0.7
GaSb-Si	0.05	CdTe-Ge	0.85		

Two-Dimensional Electron Gas (2DEG)

For n-doping concentrations in AlGaAs of about 10^{18} cm^{-3} , typical thicknesses of the electron enrichment layer are in the region 50-100 Å in GaAs. The free electrons are confined in a narrow triangular potential well in the z direction, perpendicular to the heterojunction. The wave-function of such an electron thus has Bloch-wave character only parallel to the heterostructure; one speaks of a two-dimensional electron gas (2DEG). This so-called z quantization can be described straightforwardly using the time-independent Schrodinger equation with the potential $V(z)$ and three effective mass components m_x^* , m_y^* , m_z^* . The total energy eigenvalues for such electron states quantized in the z direction



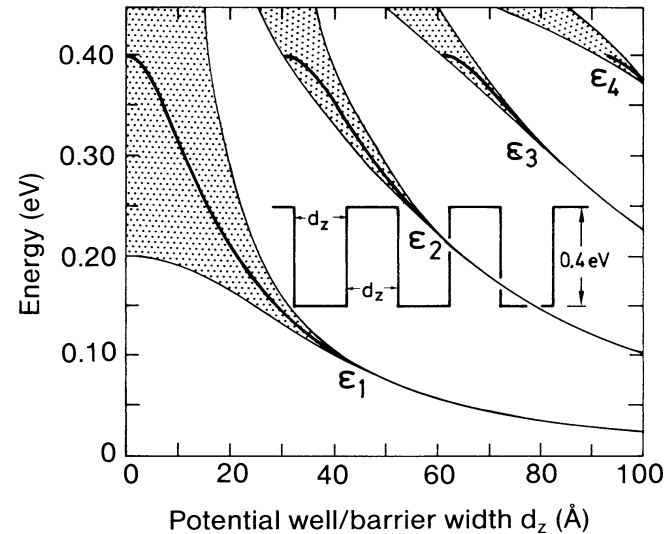
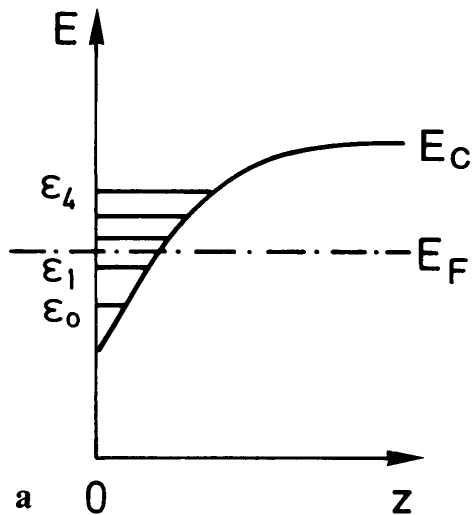
$$E_j(\mathbf{k}_{\parallel}) = \frac{\hbar^2 k_{\parallel}^2}{2m_{\parallel}^*} + \varepsilon_j \quad \text{and} \quad \varepsilon_j \simeq \frac{\hbar^2 \pi^2}{2m_z^*} \frac{j^2}{d_z^2}, \quad j = 1, 2, 3, \dots$$

These 2D subbands have a constant density of states $D(E) = dZ/dE = m_{\parallel}^*/\pi\hbar^2 = \text{const}$

$$\therefore dZ = \frac{2\pi k dk}{(2\pi)^2} \quad \text{and} \quad dE = \hbar^2 k dk/m_{\parallel}^*$$

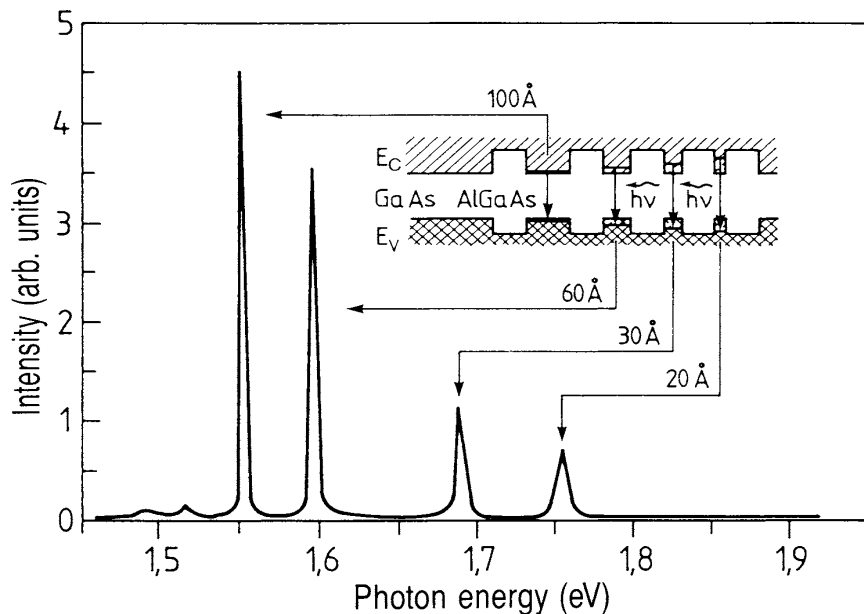
Semiconductor Superlattices

A series of layers of semiconductors I and II with different band gaps can form a *composition superlattice*. If the distance between the potential wells is so small (less than 50-100 Å) that significant overlap between the wavefunctions exists, then this leads to a broadening of the bands.



The energetically lowest subband ϵ_1 is noticeably broadened for d_z less than 50 Å, and splits off as a band. For the higher subbands, the broadening begins at even larger d_z between the potential wells.

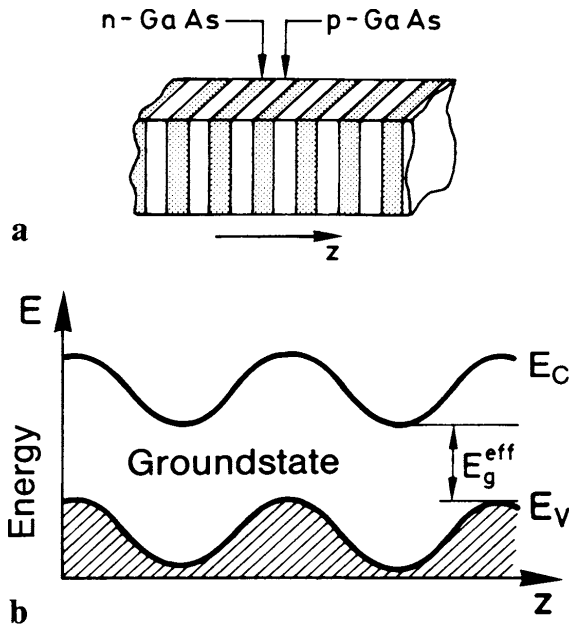
The broadening of the subbands and, in particular, the dependence of the subband energies on the spatial width of the potential wells is clearly seen in photoluminescence experiments. Photoluminescence spectroscopy is an important optical method for characterizing semiconductor heterostructures and superlattices. The semiconductor structure is illuminated with monochromatic laser light of photon energy above the band edge, thus creating electron-hole pairs. They occupy the subbands of the conduction or valence bands of the semiconductor or the corresponding excitonic states.



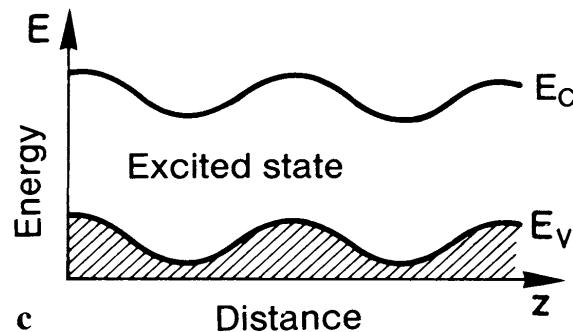
Local variations in the width of the quantum well lead to a spread in the spectral position of the emitted photoluminescence line. The energy difference between electron and hole subbands differs from the energy of the emitted photon by the amount of the exciton binding energy.

Doping Superlattice

The superlattice structure, consisting of one and the same semiconductor with the material periodically and alternately n and p doped, is called *doping superlattice*. In principle it is equivalent to a periodic sequence of p - n junctions. Because quasi intrinsic (i regions) exist between each n and p zone, these structures also have the name "nipi structures". This leads to a periodic modulation of the band edges with position.

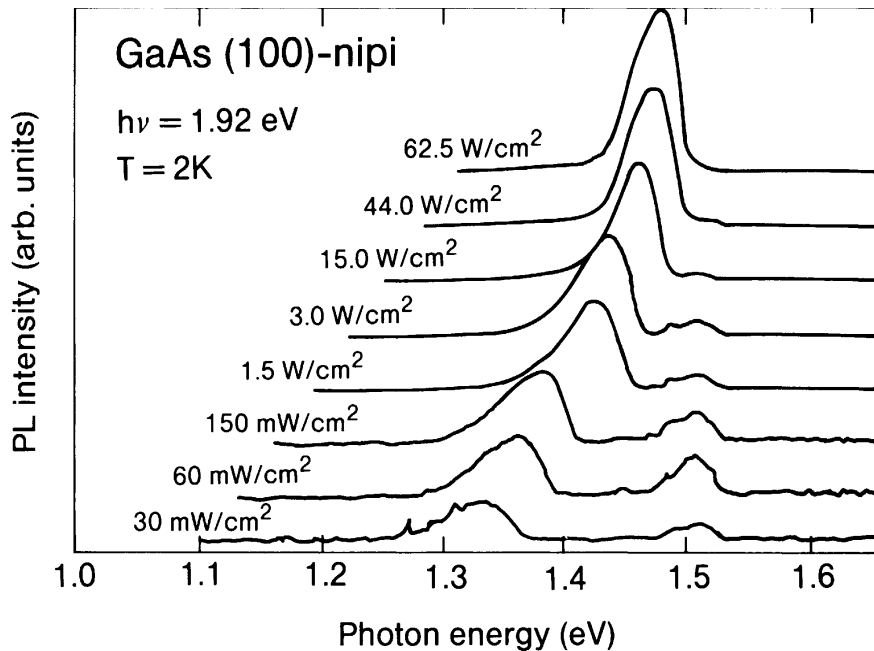


Excited free electrons are found in the minima of the conduction band, while excited holes are spatially separated and gather in the maxima of the valence band.



This spatial separation of electrons and holes is responsible for the fact that the collision rate between these two particles is drastically reduced.

Another interesting property of doping superlattices concerns the band gap. In *nipi* superlattices, the effective band gap is dependent on the density of optically excited non-equilibrium carriers. Thus the effective band gap can be optically altered. This may be demonstrated in a photoluminescence experiment in which the emission due to recombination of optically excited electrons and holes is observed as a function of laser excitation power.



Because of the relationship between band bending and space-charge density, it is immediately clear that a reduction of the space charge results in a decrease of the band bending and therefore a flattening out of the band modulation. The effective band gap becomes larger.

Sufficiently energetic excitation of electrons and holes by irradiation with light reduces the space charge and also the band modulation.

Problems

1. A semiconductor with a band gap energy E_g of 1eV and equal hole and electron effective masses $m_e^* = m_h^* = m_0$ (m_0 is free electron mass) is p -doped with an acceptor concentration of $p = 10^{18} \text{ cm}^{-3}$. The acceptor energy level is located 0.2eV above the valence band edge of the material.
 - a) Show that intrinsic conduction in this material is negligible at 300 K.
 - b) Calculate the conductivity σ of the material at room temperature (300 K), given a hole mobility of $\mu_p = 100 \text{ cm}^2/\text{Vs}$ at 300 K.
 - c) Plot the logarithm of the hole concentration, $\ln p$, versus reciprocal temperature $1/T$ for the temperature range 100 to 1000 K.
2. A silicon p - n step junction diode is doped with $N_d = 10^{16} \text{ cm}^{-3}$ and $N_a = 4 \times 10^{18} \text{ cm}^{-3}$ on the n side and p side, respectively. Calculate the build-in potential, space charge width, and maximum field at zero bias at room temperature.

3. Small-gap semiconductors such as InAs ($E_g = 0.35$ eV), and InSb ($E_g = 0.18$ eV) usually exhibit surface Fermi level pinning within the conduction band (approximately 100 meV above the lower conduction band edge E_C for InSb). Plot qualitatively the band scheme (band energy versus spatial coordinate) in the vicinity of a metal contact to such a semiconductor that is highly n -doped. What is the electrical resistance behavior for both bias directions?